3.1 A point mass m slides without friction along a wire bent into a vertical circle of radius a. The wire rotates with constant angular velocity $\mathbf{\Omega}$ about the vertical diameter, and the apparatus is placed in a uniform gravitational field $\mathbf{g}$ parallel to the axis of rotation.

(*a*) Construct the lagrangian for the point mass using the angle θ (angular displacement measured from the downward vertical) as generalized coordinate.

(*b*) Show that the condition for an equilibrium circular orbit at angle θ_0 is $\cos\theta_0 = g/a\Omega^2$. Explain this result by balancing forces in a co-rotating coordinate system.

(*c*) Demonstrate that this orbit is stable against small displacements along the wire and show that the angular frequency of oscillation about the equilibrium orbit is given by $\omega^2 = \Omega^2 \sin^2\theta_0$. *Hint:* Write $\theta = \theta_0 + \eta(t)$, where $\eta(t)$ is a small quantity.

(*d*) What happens if $a\Omega^2 < g$?

3.6 An inextensible massless string of length l passes through a hole in a frictionless table. A point mass m at one end moves on the table and a point mass m hangs from the other end.

(*a*) Write the lagrangian for this system.

(*b*) Under what condition will the hanging mass remain stationary?

(*c*) Starting from the situation in part (*b*), the hanging mass is pulled down slightly and released. State clearly what is conserved during this process.

(*d*) Compute the subsequent motion of the hanging mass.

3.7 The point of suspension of a pendulum m is allowed to move in the horizontal direction. Two springs of force constant $\frac{1}{2}k$ exert a net restoring force $-kx$ on the point of suspension.

(*a*) Use the coordinates x (the displacement of the point of support) and θ (the angular displacement of m from the vertical) to write the lagrangian and the equations of motion.

(*b*) Assuming small oscillations, show that this system is equivalent to a simple pendulum of length $l + mg/k$.

3.8 A point mass m slides without friction inside a surface of revolution $z = \alpha \sin(r/R)$ whose symmetry axis lies along the direction of a uniform gravitational field $\mathbf{g}$. Consider $0 < r/R < \frac{1}{2}\pi$.

(*a*) Construct the lagrangian $L(r, \phi; \dot r, \dot\phi)$ and compute the equations of motion for the generalized coordinates r and ϕ.

(*b*) Are there stationary horizontal circular orbits?

(*c*) Which of these orbits is stable under small impulses along the surface transverse to the direction of motion?

(*d*) If the orbit is stable, what is the frequency of oscillation about the equilibrium orbit?

3.9 Repeat Prob. 3.8 for a surface of revolution $z = \alpha(1 - e^{-r/R})$ whose symmetry axis lies along the direction of a uniform gravitational field $\mathbf{g}$.

3.10 The curve traced in space by a point on a rolling wheel is called a *cycloid*.

(*a*) Show that the equation of the cycloid passing through the origin and generated by a wheel of radius a rolling underneath the x axis is

$$x = a(\theta - \sin\theta) \qquad \text{and} \qquad y = a(1 - \cos\theta)$$

where the y axis is chosen as in Fig. 17.2. Show that the distance s measured along the cycloid from the first minimum is given by

$$s = -4a\cos\tfrac{1}{2}\theta$$

3.15 Consider the problem discussed in Sec. 16 of a bead of mass m moving without friction on a horizontal circular hoop that rotates with constant angular velocity ω about an axis passing through the wire and perpendicular to its plane (see Fig. 16.2).

(*a*) Compute the reaction force of the wire on the bead by means of a Lagrange multiplier and Lagrange's equations.

(*b*) Interpret the result by going to a co-rotating coordinate system whose origin lies at the *center* of the circular hoop