earth for any value of r.

3.13 (*a*) Express the lagrangian of a simple pendulum in terms of the cartesian variables $x = l \cos \theta$ and $y = l \sin \theta$.

(*b*) Use the holonomic constraint $x^2 + y^2 = l^2$ to rewrite L explicitly in terms of x and $\dot{x}$. Obtain the corresponding dynamical equation and reconcile it with Eq. (16.5).

(*c*) Write out Eq. (18.10) in terms of the variations δx and δy. Eliminate δy in terms of δx with the holonomic constraint and hence obtain the same dynamical equation for the variable $x(t)$.

(*d*) As an alternative procedure, use a lagrange multiplier λ to incorporate the constraint. Hence obtain a set of equations for $x(t)$, $y(t)$, and λ. Compare with the treatment of Eq. (19.13) in two-dimensional polar coordinates.

3.14 A particle of mass m suspended by a massless string of length l is stationary in a gravitational field $\mathbf{g}$. It is struck an impulsive horizontal blow, giving it an initial angular velocity ω.

(*a*) Introduce a lagrange multiplier and prove the following statements:

1. If $l\omega^2 < 2g$, the tension τ does not vanish and the particle does not reach the horizontal.
2. If $2g < l\omega^2 < 5g$, the particle passes the horizontal and the string becomes slack before the particle comes to rest.
3. If $5g < l\omega^2$, the string always remains taut and the particle executes periodic circular motion.

(*b*) Discuss the role of the tension in the string by showing how these results are changed if the string is replaced by a rigid massless rod.

reduced to explicit integrals. Does the ladder ever lose contact...

3.19 A point mass m is constrained to move without friction on an arbitrary, fixed two-dimensional surface in the absence of external forces. The surface is described by a set of generalized coordinates (q^1, q^2) such that the square of the distance $d\mathbf{s} \cdot d\mathbf{s}$ between two infinitesimally close points on the surface is given by

$$d\mathbf{s} \cdot d\mathbf{s} = ds^2 = \sum_{i=1}^{2} \sum_{j=1}^{2} g_{ij}(q^1, q^2)\, dq^i\, dq^j$$

where the *metric tensor* $g_{ij}(q^1, q^2) = g_{ji}(q^1, q^2)$ is symmetric and depends on position.

(*a*) Construct the lagrangian. Show that the equations of motion for the particle are given by

$$\sum_j g_{ij} \frac{d^2 q^j}{dt^2} + \frac{1}{2} \sum_{j,k} \left(\frac{\partial g_{ij}}{\partial q^k} + \frac{\partial g_{ik}}{\partial q^j} - \frac{\partial g_{jk}}{\partial q^i} \right) \frac{dq^j}{dt} \frac{dq^k}{dt} = 0 \qquad i = 1, 2$$

Introduce the inverse $(g^{-1})_{ij} \equiv g^{ij}$ of the metric tensor, and derive the equivalent equations of motion

$$\frac{d^2 q^i}{dt^2} + \sum_{j,k} \Gamma^i_{jk} \frac{dq^j}{dt} \frac{dq^k}{dt} = 0 \qquad i = 1, 2$$

where the *affine connection* is defined by

$$\Gamma^i_{jk} \equiv \frac{1}{2} \sum_m g^{im} \left(\frac{\partial g_{mj}}{\partial q^k} + \frac{\partial g_{mk}}{\partial q^j} - \frac{\partial g_{jk}}{\partial q^m} \right)$$

(*b*) Show that the curves of minimum distance between two points on the surface, the *geodesics*, are given by

$$\frac{d^2 q^i}{d\tau^2} + \sum_{j,k} \Gamma^i_{jk} \frac{dq^j}{d\tau} \frac{dq^k}{d\tau} = 0 \qquad i = 1, 2$$

where $0 \leqslant \tau \leqslant 1$ is a uniform parametrization of the distance along the curve.

(*c*) Show that $\mathbf{v}^2 \equiv v^2 = \text{const}$ for any motion on this surface. For a given trajectory of the particle passing through two points on the surface a distance l apart, prove that we can take $\tau = (v/l)t = \text{const} \times t$. Hence conclude that the equations and curves in *a* and *b* are identical. *Note:* These observations form one of the starting points for the theory of general relativity.